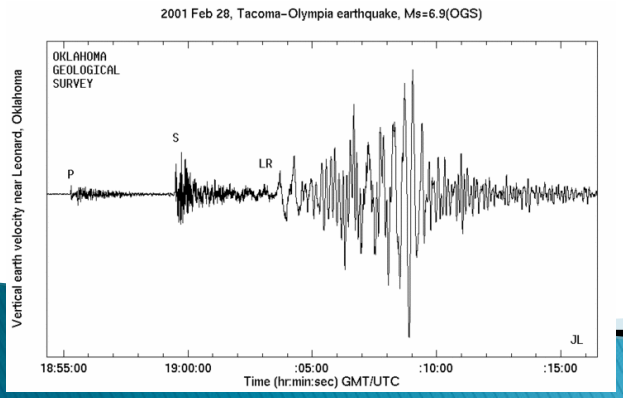
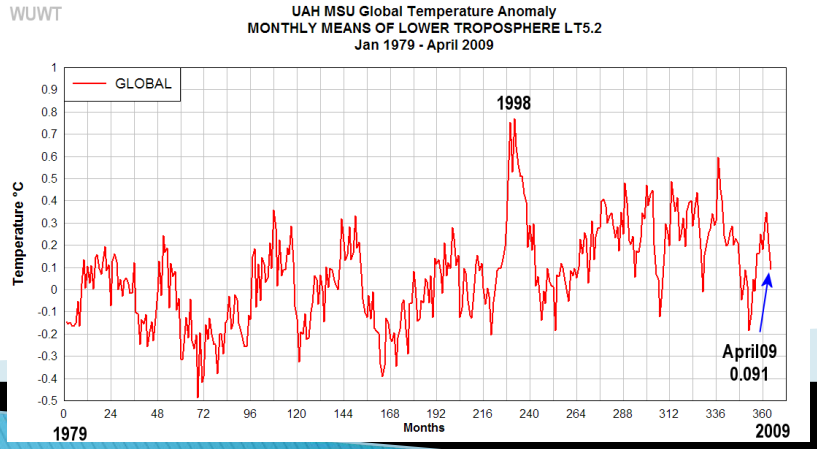
Wavelets, detection of singularities and image processing

Eugenio Hernández
Universidad Autónoma de Madrid

Universidad Carlos III de Madrid
23 de abril de 2015

1.1. Signals and singularities





Some are continuous (left), some are discrete (right)

Continuous data must be discretize to manipulate in computers

1.2. Signals and singularities

SHANNON-WHITTAKER THEOREM.

If f is a continuous signal in $L^1(\mathbb{R})$ such that its Fourier transform $\mathcal{F}f$ has support in $[-\frac{T}{2}, \frac{T}{2}]$, $T > 0$, then

$$f(x) = \sum_{k=-\infty}^{k=\infty} f(\frac{k}{T}) \frac{\sin \pi(Tx - k)}{\pi(Tx - k)}.$$

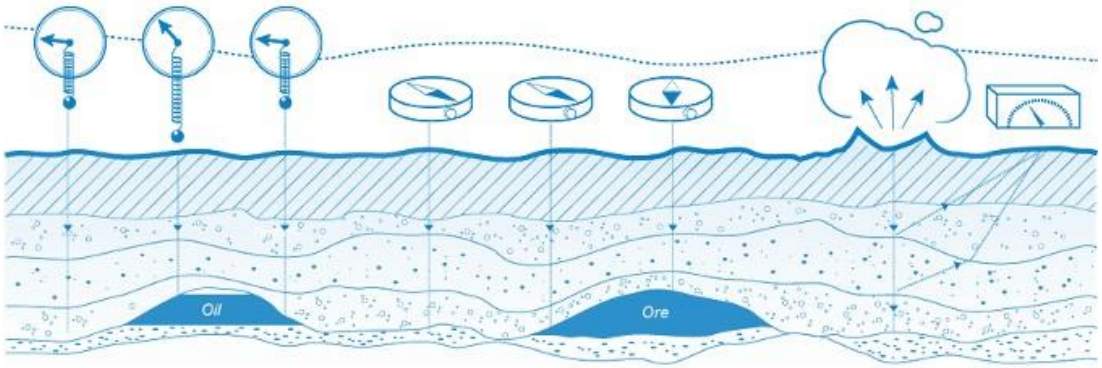
A good discretization interval for f is 1/T or smaller

FOURIER TRANSFORM.

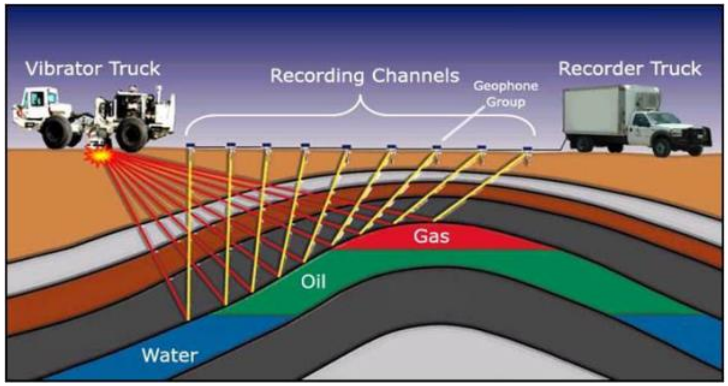
$$\mathcal{F}(f)(\xi) = \int_{\mathbb{R}} f(x)\, e^{-2\pi i x \xi}\, dx\,.$$

All signals are treated as vectors $\mathbf{x}$ in $\mathbb{R}^N$.

1.3 How are hydrocarbon fields discovered?



Gravity,
Magnetic
Seismic

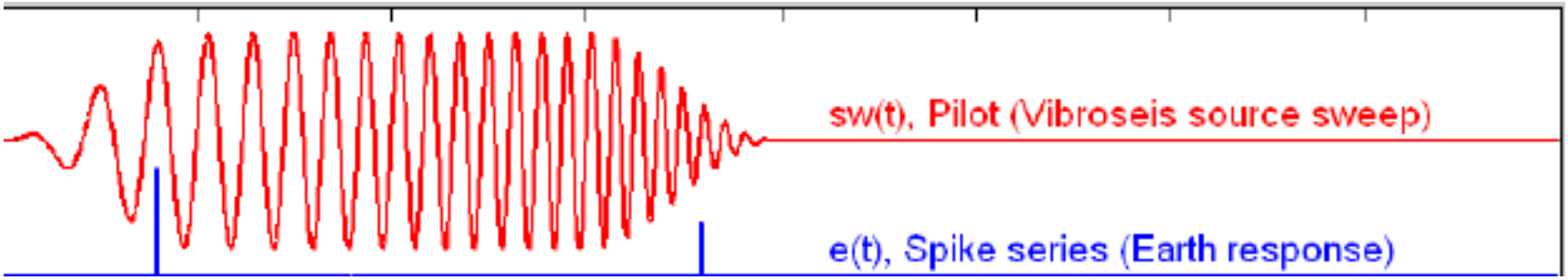


Vibration techniques

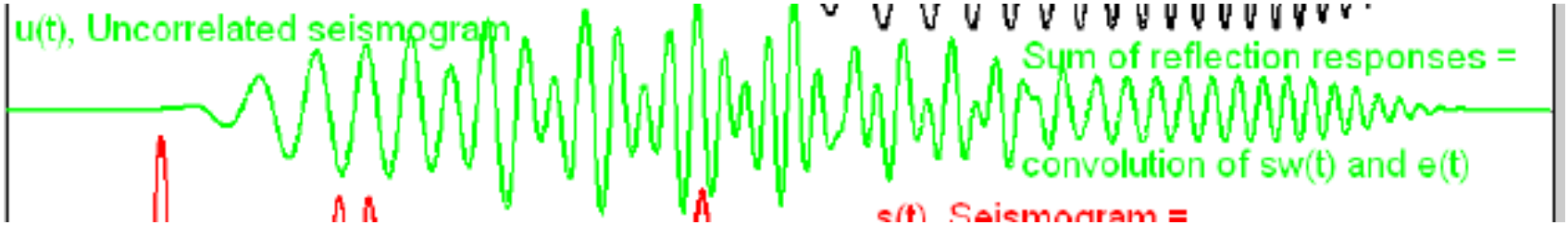
There are tehniques to identify accumulations of hydrocarbons. But, the only way to know for sure is to drill a well.

1.4. How are hydrocarbons fields discovered?

Vibration wave emited by vibration source.



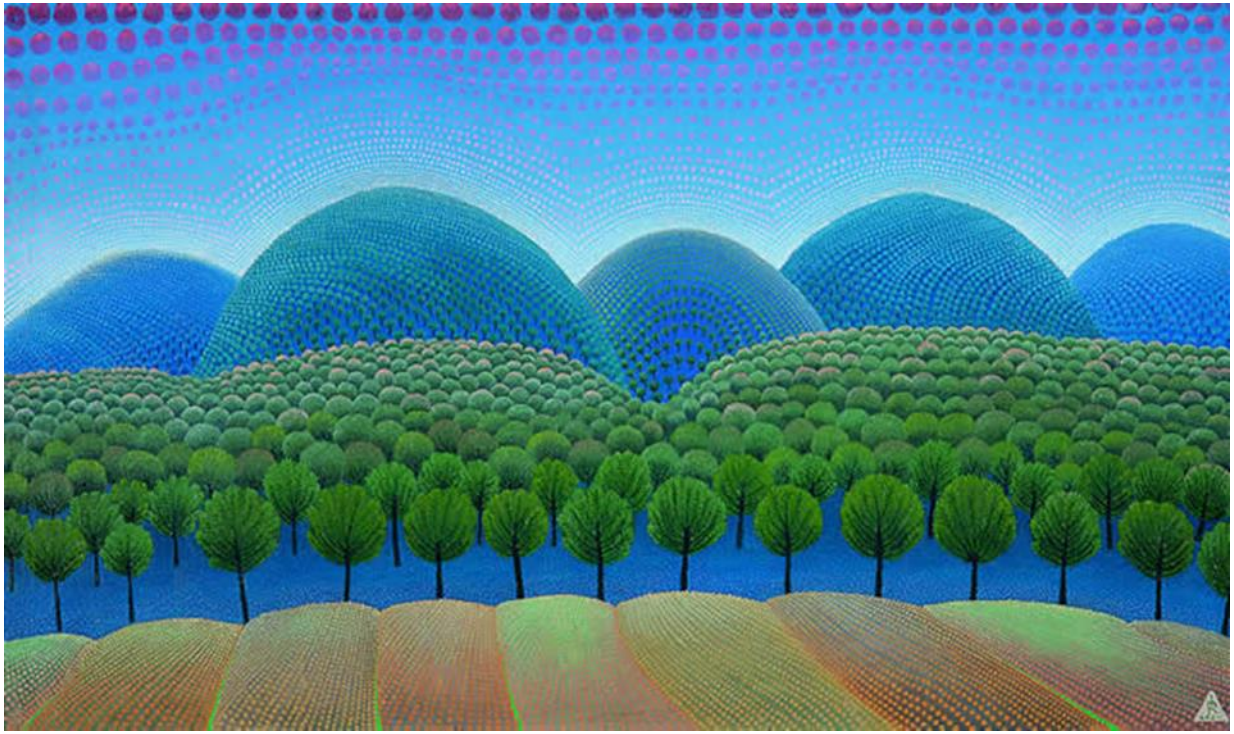
Reflection response recorded by geosensors.



Singularities may be an indication of hydrocarbons.

2.1: Images: compression and edge detection

Naive art and image representation

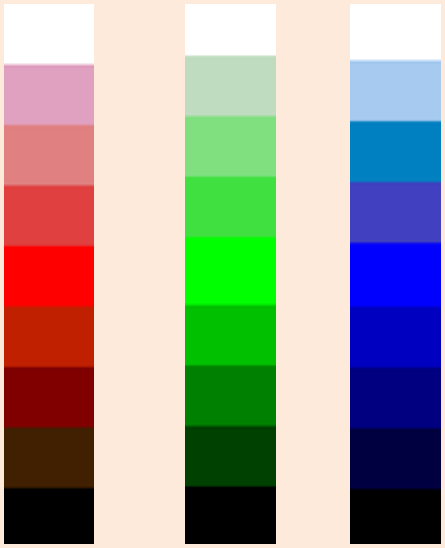


Ivan Rabuzin picture (In the Hills, 1960, Croatian Museum of Modern Art) is formed by colored dots. One can imagine that these dots represent the color of each pixel of an image for digital representation.

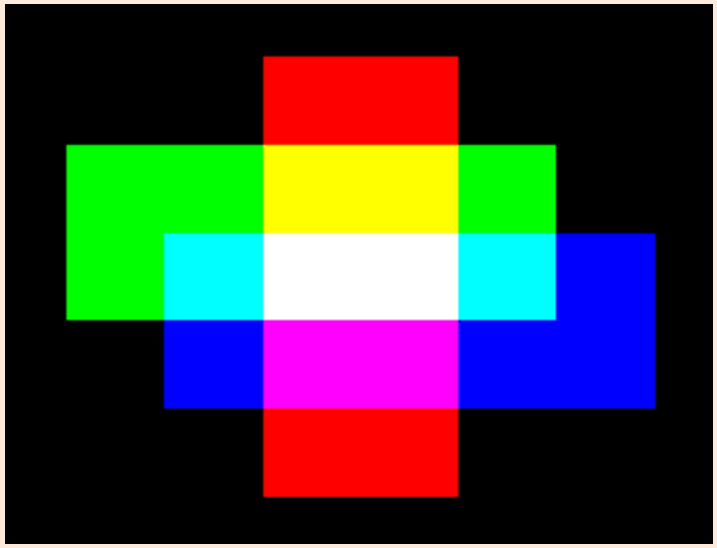
2.2: Images: compression and edge detection

Image representation: the RGB system to represent color

The color of each pixel is represented by an appropriate combination of each one of the primary colors RED, GREEN and BLUE.



Different intensities of primary colors

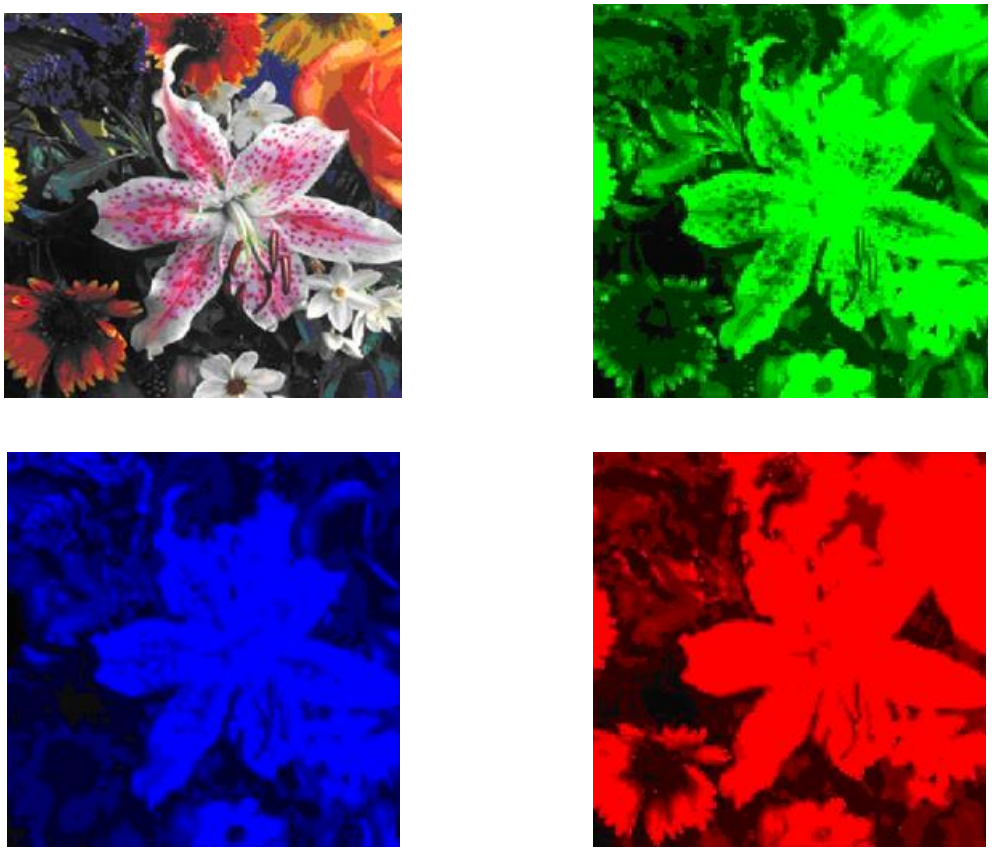


Colors obtained by superposition of primary colors

Each color intensity is represented by a number between 0 and 255 (8 bits)

2.3: Images: compression and edge detection

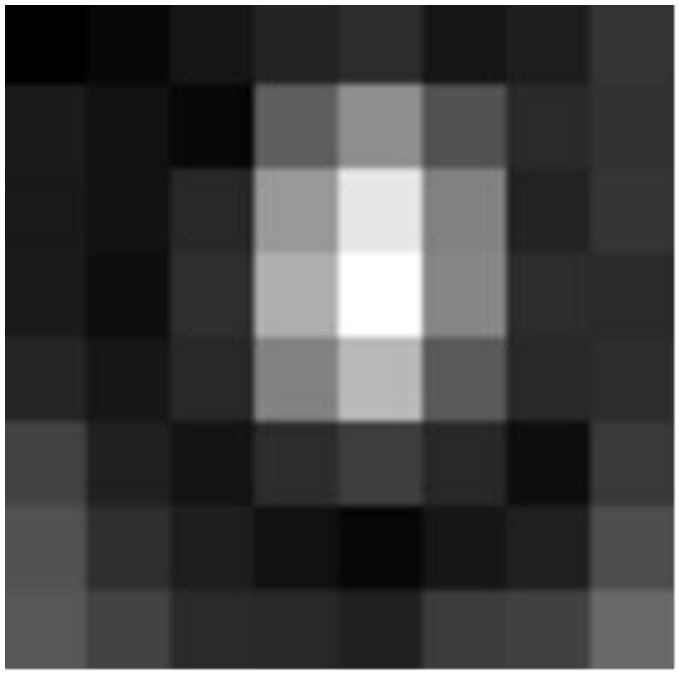
The RGB system to represent color



Primary colors distribution in a flower picture

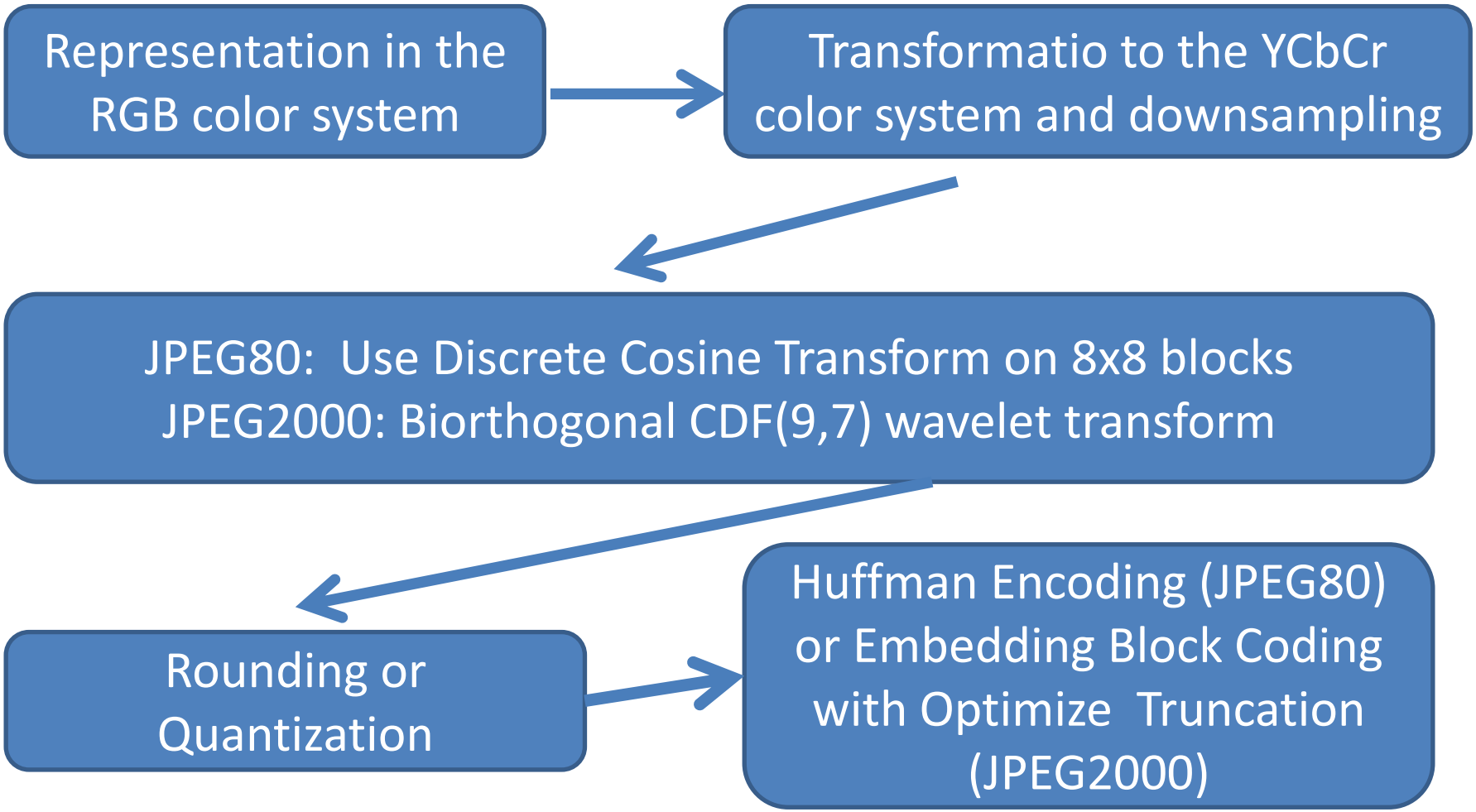
2.4. Grey scale representation of an image

$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$



A grey scale representation of an image is an array of numbers of size MxN.

2.4. JPEG80 and JPEG2000
Joint Photographic Experts Group

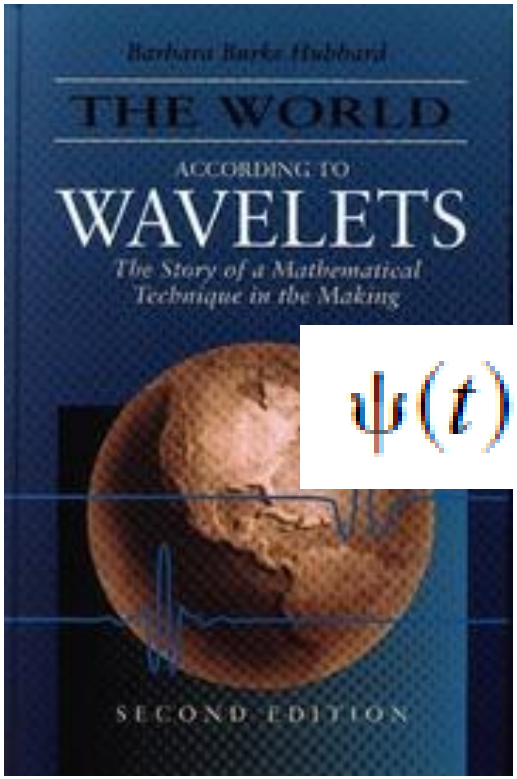


2.5. Recovery of a compressed image of flower with JPEG2000

Original image

Compressed image with JPEG2000. Compression rate of 15/100

3.1 History



- Wavelet originated in geophysics around 1982 for the analysis of seismic signals.

$$\psi(t) = \pi^{-1/4} e^{-i\omega_0 t} e^{-t^2/2} \qquad \omega_0 \geq 5.$$

- Continuous wavelet transform formalized by J. Morlet, A. Grossman and P. Goupillaud to study signals obtained from oil surveys (1984).

3.2 History

Y. Meyer realized the importance of the work of Morlet, Grossman and Goupillaud and and its relation to Harmonic Analysis (1985)

S. Mallat, with the help of Y. Meyer, described the concept of MRA, based in optics. (1987)

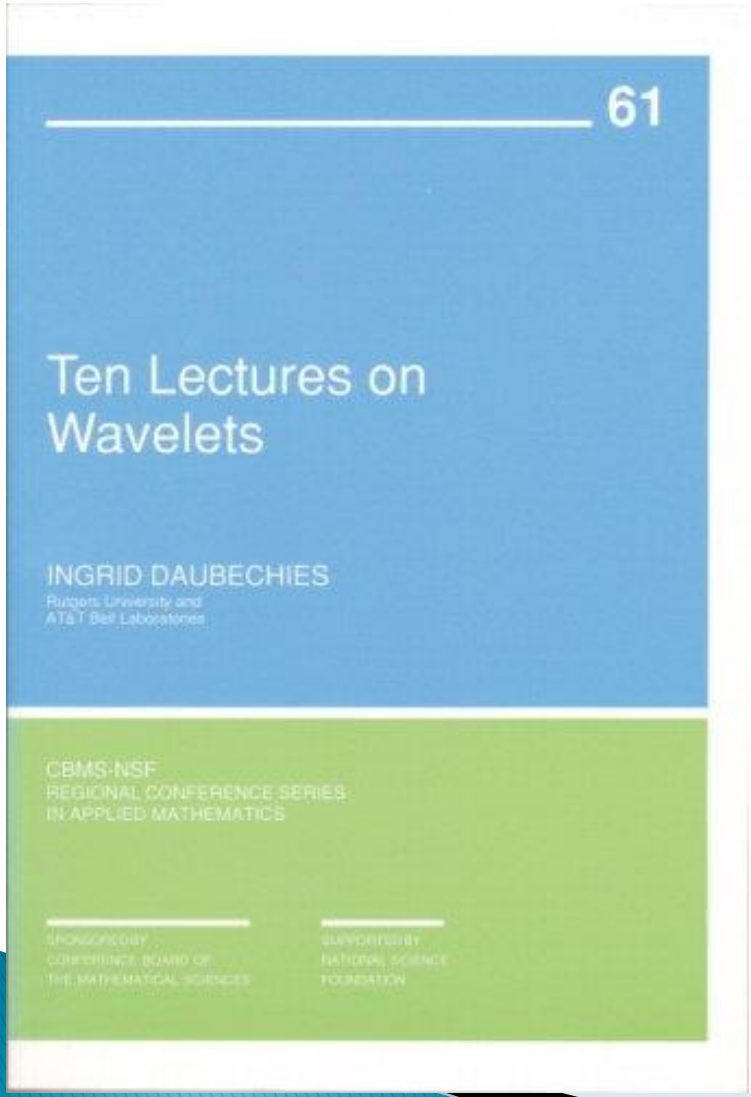
I. Daubechies developed compactly supported wavelets for applications in idetection of singularities, denoising and image compression.

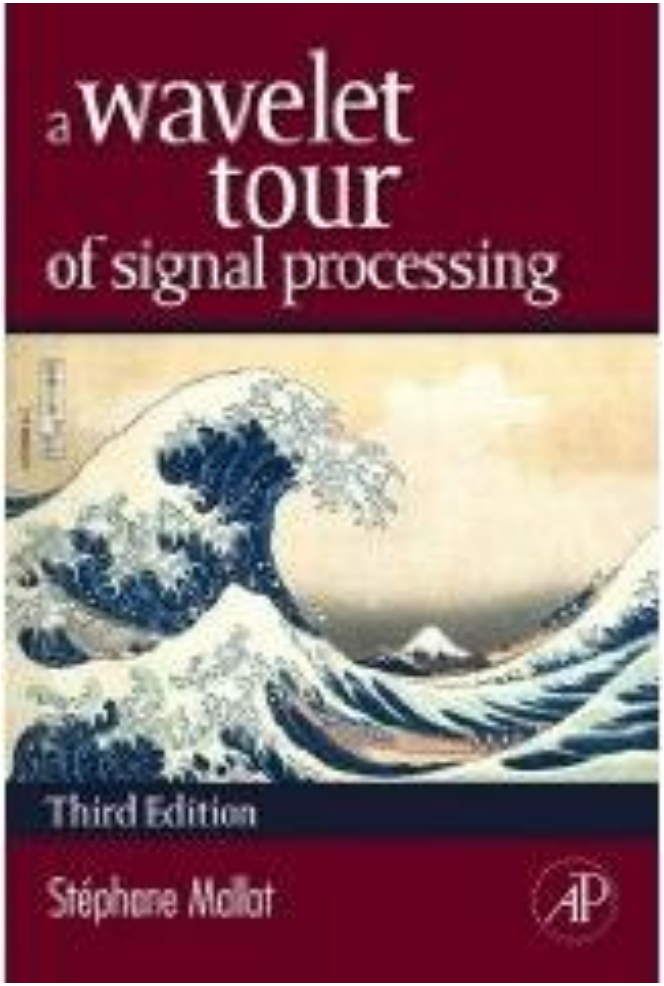
3.3 History

Development of wavelet packets by R. Coifman, Y. Meyer and V. Wickerhauser (1992) to be able to choose a wavelet transform better adapted to te features of a signal or image.

A. Cohen, I. Daubechies and J.C. Feauveau developed biorthogonal wavelets with compact support (1992) latter becoming standard in JPEG2000.

4.1 Bibliography





4.2 Bibliograhpy